Answer 42

42. For what negative value of k, the system of equations below has no solutions?

2x + ky = 5
kx + (k + 4)y = 7

A) -2
B) -5
C) -4
D) -7
E) -1

One way to solve this is to realize that if a system of equations has no solutions, the lines must be parallel and have different y intercepts. So, the equations can be manipulated into slope-intercept form:

$$2x + ky = 5 \quad ky = -2x + 5 \quad y = \frac{-2}{k}x + \frac{5}{k}$$

And

$$kx + (k + 4)y = 7 \quad (k + 4)y = -kx + 7 \quad y = \frac{-k}{(k + 4)}x + \frac{7}{(k + 4)}$$

At this point, it seems very likely that the y intercepts of those two equations are different, so now we just need to make sure the slopes are the same to force the lines to be parallel.

$$\frac{-2}{k} = \frac{-k}{(k + 4)}$$

$$-2(k + 4) = -k^2$$

$$-2k - 8 = -k^2$$

Simplify as a quadratic:

$$k^2 - 2k - 8 = 0$$

This factors into

$$(k - 4)(k + 2)$$

So, there are two roots, k = 4 and k = -2. The question asks for a negative value of k, so the answer is A.